SPEEDFACTORIES: GLOBAL DUAL SOURCING UNDER CORRELATED DEMAND

OR Seminar Series at The University of Exeter, College of Engineering, Mathematics and Physical Sciences

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Our motivation: Adidas' SpeedFactory

- Adidas sells 301 million shoes each year, mainly made in Asia.
- Aims to open SpeedFactories in Germany and the US to produce 1 million shoes a year.
- The localised factory offer shorter lead-times (5 hours, compared to many weeks from the Asian supply chain). This short lead-time allows:
    - supply to better match demand, simplifying supply chains, reducing inventory, storage, transportation, and obsolescence
    - customised shoes to be offered
- Robots also help to increase productivity by up to 30%

Shotter and Whipp (2016)

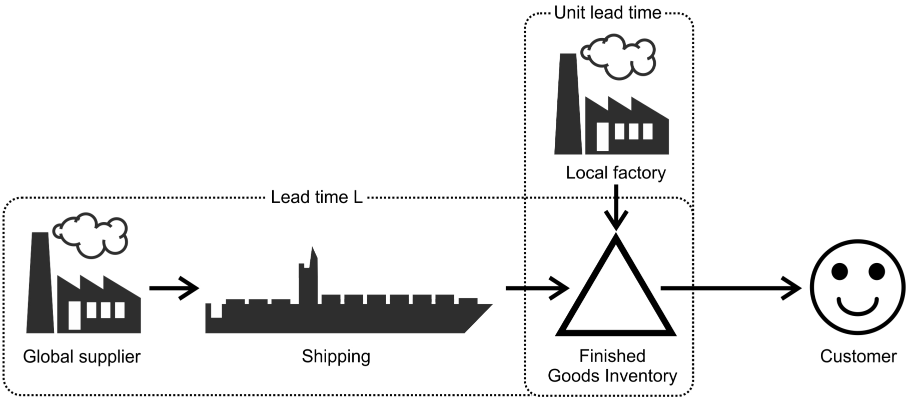
Overview

We consider three supply chain models

- *Local sourcing*, where all of the demand is produced locally with unit lead-time.
- *Full off-shoring*, where all of the demand is purchased from a global supplier with a lead time of L periods.
- *Dual sourcing*, where 80% of the demand is sourced from the global supplier and 20% produced locally[1].

[1] We assume a fixed percentage of product is re-shored to facilitate the exposition of this short conference paper. We refer interested readers to Boute et al. (2018) for more information on the optimal proportion to re-shore.

Our supply chain replenishes inventory from a global supplier and/or a local factory



The first order auto-regressive (AR(1)) demand process

- Ali et al. (2012) found that 46% of 1798 SKU's in a European retailer belonged to, or were sub-sets of, AR(1) demand.
- The AR(1) demand process is given by

$$d_t = \mu + \rho \left(d_{t-1} - \mu \right) + \epsilon_t, \tag{1}$$

  Box et al. (2008), where
  - d_t is the demand in time period t
  - μ is the mean demand
  - ρ is the auto-correlation coefficient of demand,
  - ϵ_t is a stochastic independent and identically distributed (i.i.d.) random variable with zero mean and variance σ^2
- We assume that ϵ_t is normally distributed and take a linear approach to our analysis. This means that all state variables will also be normally distributed.

The AR(1) demand process

All AR(1) demands are mean reverting,

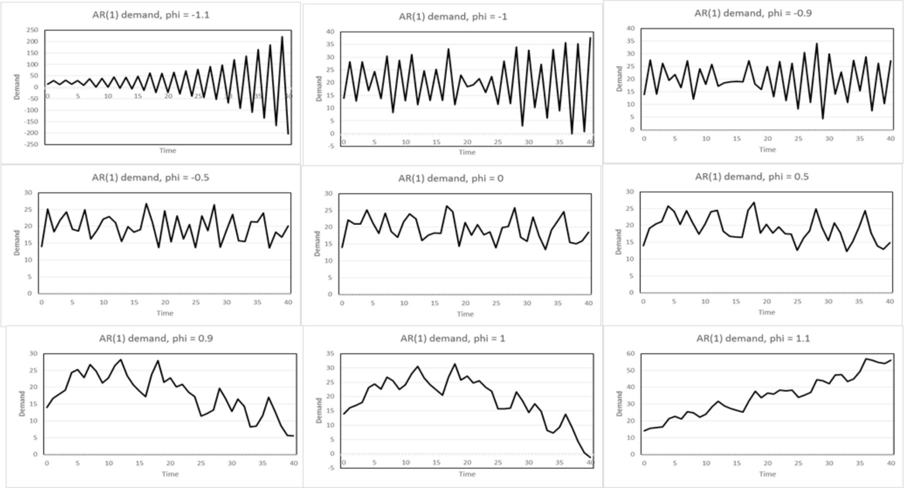
- positive correlation ($0 < \rho < 1$) is associated with demand processes with momentum
- negative correlation ($-1 < \rho < 0$) is characterized by high frequency, period-to-period oscillations
- zero correlation ($\rho = 0$) degenerates into an independently and identically distributed random demand process.

Box et al. (2008) shows the demand variance is

$$\sigma_d^2 = \frac{\sigma^2}{1 - \rho^2}. \tag{2}$$

Remark. $0 \leq \rho \leq 0.7$ in many real AR(1) demand patterns.

Example AR(1) demand processes



Purchasing costs

To purchase product from the global external supplier a per unit purchase cost is incurred,

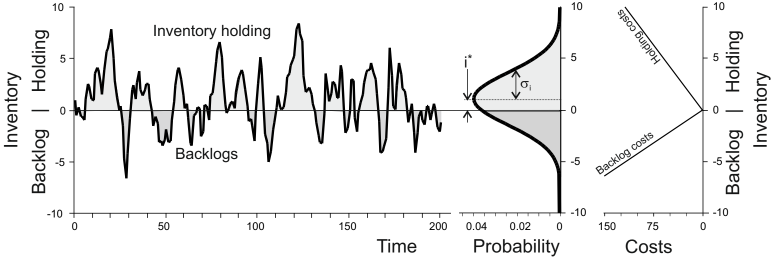
$$C_t^p = pq_t^g, \tag{3}$$

where p is the total landed unit purchasing cost and q_t^g is the quantity purchased from the global supplier in period t.

Inventory costs

Per period, per unit, inventory holding (h) and backlog costs (b) exist:

$$C_t^i = h[i_t]^+ + b[-i_t]^+. \tag{4}$$



Using standard newsvendor techniques (Churchman et al., 1957) a safety stock of

$$i^\star = \sigma_i z_i;\;\; z_i = \Phi^{-1}\left[b/(b+h)\right] \tag{5}$$

results in the minimal long-run average per period inventory cost

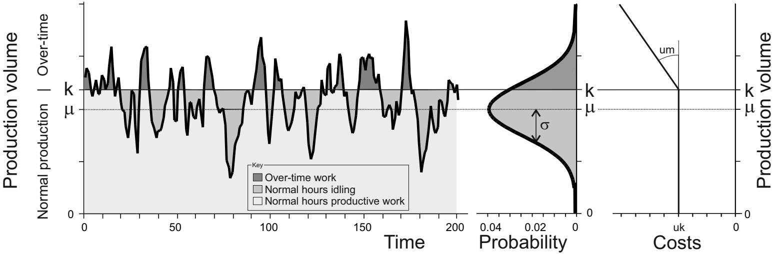
$$C_i^\star = \sigma_i (h+b)\phi\left[z_i\right]. \tag{6}$$

Notation: $[x]^+ = \max[x, 0]$, and $\{\phi[\cdot], \Phi^{-1}[\cdot]\}$ is the p.d.f., and inverse of the c.d.f., of the standard normal distribution respectively.

Production costs at the local factory

A per period production cost C_t^q, with a nominal hours unit cost of u and flexible overtime at unit cost um, where m is the overtime multiplier:

$$C_t^q = uk + um[q_t - k]^+. \tag{7}$$



Hosoda and Disney (2012) show the optimal capacity $k^\star$ is given by

$$k^\star = \gamma\mu + \sigma_q z_q; \quad z_q = \Phi^{-1}\left[(m-1)/m\right]. \tag{8}$$

The associated minimal long-run average production costs is

$$C_q^\star = u\gamma\mu + um\sigma_q\phi[z_q]. \tag{9}$$

Replenishment policies, variance and costs

- The inventory costs, C_i^*, and the capacity costs, C_q^*, are functions of the standard deviation of the inventory levels and order quantities.
- These, in turn, are influenced by the replenishment policy, forecasting systems, and lead-times.
- Next, we define these factors for each of our three models.
- Then we will determine the variance expressions before identifying expressions for the expected costs.

The replenishment policies: Full global off-shoring

The inventory optimal order-up-to (OUT) policy, Li et al. (2014), places replenishment orders for the global supplier in the off-shore case,

$$q_t^g = \hat{d}_{t+L|t} + \left(i_g^\star - i_t^g\right) + \sum_{j=1}^{L-1}\left(\hat{d}_{t+j|t} - q_{t-j}^g\right). \tag{10}$$

$\hat{d}_{t+L|t}$ is the minimum mean squared error forecast of the demand in period $t+L$ conditional upon the information available at time t (Box et al., 2008).

The replenishment policies: Local sourcing only

When the demand is replenished from the *capacitated* local factory with unit lead time ($L = 1$) we determine production orders with the proportional order-up-to (POUT) policy (Disney et al., 2016):

$$q_t^l = \hat{d}_{t+1|t} + (1 - \alpha)\left(i_l^\star - i_t^l\right). \tag{11}$$

When $0 \leq (1 - \alpha) < 1$ a degree of production smoothing is present that can reduce the bullwhip effect and capacity costs.

The replenishment policies: Dual sourcing

The global supplier is given a replenishment order equal to 80% of the conditionally expected demand in the period after the lead time (minus one period);

$$q_t^s = \hat{d}_{t+L,t} - 0.2\mu = \rho^L(d_t - \mu) + 0.8\mu \tag{12}$$

The local production orders control the local FGI with,

$$q_t^d = \hat{d}_{t+1,t} - 0.8\mu - q_{t-L+1}^s + (1-\alpha)\left(i_d^\star - i_t^d\right) \tag{13}$$

Variances and costs: Full global off-shoring

Disney and Lambrecht (2008) provide the variance of the finished goods inventory has

$$\sigma_{i,g}^2 = \frac{\sigma^2 \left(L\left(\rho^2-1\right)+\rho\left(\rho^L-1\right)\left(\rho^{L+1}-\rho-2\right)\right)}{(\rho-1)^3(\rho+1)}. \quad (14)$$

The expected purchasing cost in each period is $C_p^\star = p\mu$.

The total cost per period for full global off-shoring is

$$C_T^\star|_g = p\mu + \sigma_{i,g}(h+b)\phi\left[\Phi^{-1}\left[b/(b+h)\right]\right]. \quad (15)$$

Remark. Under positively correlated demand, $\rho > 0$, $\sigma_{i,g}^2$ is increasing in L, which together with (15) mean the full off-shoring costs increase in L.

Variances and costs: Local sourcing

Chen and Disney (2007) provide the required variance expressions: The local order variance is

$$\sigma_{q,l}^2 = \frac{\sigma^2\left(\left(\alpha^2+\alpha-2\right)\rho-2\alpha\rho^2+\alpha+2\rho^3-1\right)}{\left(\alpha+1\right)\left(\rho^2-1\right)\left(1-\alpha\rho\right)}. \tag{16}$$

and the variance of the FGI is

$$\sigma_{i,l}^2 = \frac{\sigma^2}{1-\alpha^2}. \tag{17}$$

Remark. The local inventory variance is independent of the auto-regressive parameter of demand.

The total expected costs in the local sourcing only setting:

$$C_T^\star|_l = \sigma_{i,l}(h+b)\phi\left[\Phi^{-1}\left[b/(b+h)\right]\right]+u\mu+um\sigma_{q,l}\phi[\Phi^{-1}\left[(m-1)/m\right]]. \tag{18}$$

Dual sourcing variances: The variance of the local production

When the POUT policy is used to set local production targets to control the FGI in the dual sourcing setting, the variance of production orders is given by:

$$\sigma^2_{q,d}|_{POUT} = \sigma^2\left(\rho^L\left(\frac{\rho^L}{\rho^2-1} - \frac{2(\alpha-1)\alpha^{L-1}}{\alpha\rho-1}\right) - \frac{\left(\alpha^2+\alpha-2\right)\rho - 2\alpha\rho^2 + \alpha + 2\rho^3 - 1}{(\alpha+1)\left(\rho^2-1\right)(\alpha\rho-1)}\right). \tag{19}$$

Dual sourcing variances: Inventory variance

The variance of the FGI maintained by the POUT policy in the dual sourcing setting is

$$\sigma^2_{i,d}|_{POUT} = \frac{\sigma^2}{1-\alpha^2}, \tag{20}$$

Remark. The dual sourcing inventory variance is independent of the demand auto-correlation.

Costs in the dual sourcing setting

The total expected costs in the dual sourcing setting is

$$C_T^{\star}|_d = \sigma_{i,d}(h+b)\phi\left[\Phi^{-1}\left[b/(b+h)\right]\right] + 0.8p\mu + 0.2u\mu + \\ um\sigma_{q,d}\phi[\Phi^{-1}\left[(m-1)/m\right]]. \quad (21)$$

Proposition 1: The POUT policy α that minimises the total cost, $C_T^\star$, is the inverse solution to $\lambda_\alpha^\star = \sigma_q \Big/ \left(\sigma_q - \sigma_i \left(\frac{d\sigma_q^2}{d\alpha} \Big/ \frac{d\sigma_i^2}{d\alpha}\right)\right)$

Proof The both total costs can be written as

$$C_T^\star = \varphi(\sigma_i + \lambda(\sigma_q - \sigma_i)) + A, \tag{22}$$

where the weighting factor, $\lambda = um\phi[z_q]/\varphi$, the scaling factor $\varphi = (h+b)\phi[z_i] + um\phi[z_q]$ and A are other terms in the cost function that are not influenced by α. The derivative of (22) with respect to α is

$$\frac{dC_T^\star}{d\alpha} = \varphi\left(\frac{(1-\lambda)\frac{d\sigma_i^2}{d\alpha}}{2\sigma_i} + \frac{\lambda\frac{d\sigma_q^2}{d\alpha}}{2\sigma_q}\right). \tag{23}$$

From (23), the necessary first order optimality condition for $\alpha^\star$ is

$$\frac{(\lambda-1)\frac{d\sigma_i^2}{d\alpha}}{2\sigma_i} = \frac{\lambda\frac{d\sigma_q^2}{d\alpha}}{2\sigma_q}. \tag{24}$$

Rearranging (24) for λ provides the stated inverse function for α. $\square$

Remarks on Proposition 1

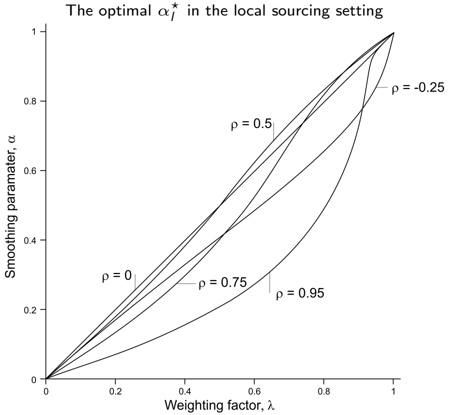
After substitution of the relevant expressions for the standard deviation of the orders and inventory levels and their derivatives into the expression for $\lambda^{\star}_{\alpha}$, algebra reveals the relationship between $\lambda^{\star}_{\alpha}$ and α. The following insights emerge:

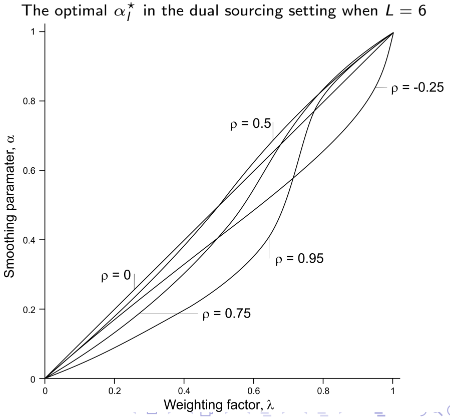
- For each weighting factor, λ, there is a unique optimal α.
- In general, for AR(1) demand, this is a high order transcendental expression with no known solution.
- For i.i.d. demand the optimal $\alpha^{\star} = \lambda$ in both the near-shore and dual sourcing supply chains.

Proposition 1 also allows one to use the efficient Regula Falsi method (a.k.a. the method of false position) to numerically obtain solutions for $\alpha^{\star}$.

The optimal smoothing parameter in the local sourcing POUT policy

Using the following modelling parameters: $\{\mu = 10, \sigma = 1, u = 4, m = 1.5, p = 3.8\}$, the weighting factor $\lambda = 0.554$. Note, near $\lambda = \{0, 1\}$, $\alpha^\star = \lambda$, when $\lambda < 0.5$, $\alpha^\star < \lambda$, $\alpha^\star \approx \lambda$ otherwise. Also, $\alpha_d^\star \to \alpha_l^\star$ when $L \to \infty$.





Total costs when $\rho = 0$, i.i.d. demand

Using $\mu = 10, \sigma = 1, u = 4, m = 1.5$, and $p = 3.8$:

Lead time L	Off-shore	Local production	Dual sourcing	% improvement of dual sourcing over off-shoring
2	42.482	43.277	41.677	1.89%
3	43.040	43.277	41.677	3.17%
4	43.510	43.277	41.677	4.21%
5	43.924	43.277	41.677	5.12%
6	44.299	43.277	41.677	5.92%
7	44.643	43.277	41.677	6.64%
8	44.964	43.277	41.677	7.31%

(Note: For local sourcing and dual sourcing, $\alpha^{\star} = 0.554$)

Total costs when $\rho = 0.5$

Using $\mu = 10, \sigma = 1, u = 4, m = 1.5$, and $p = 3.8$:

Lead time L	Off-shore	Local production	$\alpha_d^\star$	Dual sourcing	% improvement of dual sourcing over off-shoring
2	43.164	44.534	0.599	42.663	1.16%
3	44.409	44.534	0.588	42.863	3.48%
4	45.502	44.534	0.576	42.915	5.69%
5	46.468	44.534	0.570	42.929	7.62%
6	47.333	44.534	0.567	42.934	9.29%
7	48.112	44.534	0.566	42.934	10.76%
8	48.839	44.534	0.566	42.934	12.09%

(Note: With only local production, $\alpha_l^\star = 0.565$)

Total costs when $\rho = 0.95$

Using $\mu = 10, \sigma = 1, u = 4, m = 1.5$, and $p = 3.8$:

Lead time L	Off-shore	Local production	$\alpha_d^\star$	Dual sourcing	% improvement of dual sourcing over off-shoring
2	43.846	49.283	0.607	43.610	0.54%
3	46.313	49.283	0.569	44.469	3.98%
4	49.069	49.283	0.452	45.000	8.29%
5	52.054	49.283	0.344	45.354	12.87%
6	55.223	49.283	0.309	45.631	17.37%
7	58.543	49.283	0.294	45.866	21.65%
8	61.987	49.283	0.285	46.069	25.68%

(Note: With only local production, $\alpha_l^\star = 0.242$)

Theoretical contributions

- We extend the dual sourcing theory beyond i.i.d. demand and the constant global orders to correlated AR(1) demand and dynamic global orders
- A proportional order-up-to policy is used to determine local production orders due to the existence of capacity costs in the local factory.
- Comparing its cost performance against off-shoring or near-shoring *all* demand, we find our dual sourcing policy remains robust to the off-shore lead time and can outperforms both off-shoring and near-shoring.
- The benefit of dual sourcing increases as demand becomes more positively correlated. For example, the benefit of dual sourcing over off-shoring when $L = 6$ is 7.31% for under i.i.d. demand; for AR(1) demand with $\rho = 0.95$, this benefit increases to 25.68%.

Managerial implications

- As prices in dynamic, emerging economies are rising faster than the prices in the established economies then eventually re-shoring recently off-shored production will become competitive.
- We do not need not wait for price parity; responsive local production enables tighter inventory control which makes partial re-shoring economical even before parity is achieved.
- This desirable inventory effect is enhanced by demand correlation and long global off-shoring lead times.

Avenues for future work

- Future work could be directed towards identifying the optimal proportion of product to re-shore as in the paper we simply assume it to be 20%.
- Being a generalisation of the industrially common OUT replenishment policy, the POUT policy is easily implementable and is guaranteed to outperform the OUT policy. However, it is not the optimal dual sourcing policy which is non-linear in nature. Directing effort into understanding the optimal replenishment policy in our dual sourcing setting would be rewarding.
- One could also consider the impact of an installed capacity and overtime cost in a global supplier owned by the focal company (rather than a third party selling units at a purchase price).

Shiny App: Explore the SpeedFactory yourself at https://bullwhip.shinyapps.io/shiny/

SpeedFactory App

- What is a SpeedFactory?
- Cost parameters
- Demand process
- Replenishment policy
- Break-even analysis
- Simulation
- About this app

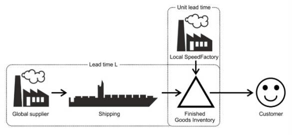
Introduction

A SpeedFactory is a small, localised, factory that is used in conjunction with a low-cost global supplier in a dual sourcing supply chain. Most of the demand, the base demand, is satisfied from the low cost global supplier, with a small proportion of the demand, the surge demand, being satisfed from the local supplier with the unit lead time.

The uncapacited global supplier manufactures and ships product with a long lead time L at a unit purchase cost into the finished goods inventory (FGI) that is owned paid for by the focal company. Inventory holding and backlog costs are applied to the FGI. The SpeedFactory incurs a production cost for the installed capacity of k, with requirements higher than k being produced, at a premium, in overtime. During periods of low requirement, the production system may be idled.

The local SpeedFactory's short lead-time maintains tight control of the FGI: a feat the global supplier with a long lead-time can not achieve. This allows in some cases, for partial reshoring of demand back to a SpeedFactory, before the off-shore price rises to parity with the local price.

Schematic of a dual sourcing supply chain with a SpeedFactory



Thank you for listening

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This presentation based upon the following articles:

Boute, R.N., Disney, S.M. and Van Mieghem, J.A., (2020), Global dual sourcing under correlated demand. Pre-prints of the 21st International Working Seminar of Production Economics, 24th-28th February, Innsbruck, AUSTRIA, 14 pages.

Boute, R., Disney, S.M., Gijsbrechts, J. and Van Meighem, J., (2020), Dual Sourcing and Smoothing under Non-Stationary Demand Time Series: Re-shoring with SpeedFactories. Accepted for publication in *Management Science*, 14th December.

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Shotter, J., L. Whipp. 2016. Robot revolution helps Adidas bring shoemaking back to Germany. https://www.ft.com/content/7eaffc5a-289c-11e6-8b18-91555f2f4fde.